Suggested solutions to the IO (BSc) resit exam on August 19, 2010
VERSION: September 2, 2010

Question 1

a) **Explain briefly the standard reason why, according to economic theory, a monopoly is bad. Also explain briefly what is meant by "rent seeking" and "X-inefficiency".**

- The *standard argument* for why a monopoly (or lack of competition more generally) is undesirable is that it gives rise to an *allocative inefficiency* — the monopoly sets a price that implies that some gains from trade are left unexploited. Some consumers in the economy have a valuation for the good that exceeds the firm's cost of producing it. Hence, if the firm and the consumer agreed to trade at some price between the consumer's valuation and the firm's cost, they both parties would gain. However, because of the monopoly (or lack of competition more generally), they do not trade.

- *Rent seeking:* In an article published in 1967, Tullock argued that the welfare losses due to a monopoly are not properly measured by the black triangle — the losses are actually bigger. Tullock argued that, to the black triangle, we should add (at least parts of) the rectangle representing the monopoly profits. Why? In Tullock's own words: "Surely we should expect that with a prize of this size dangling before our eyes, potential monopolists would be willing to invest large resources in the activity of monopolizing." That is, Tullock argued that: (i) Firms will lobby or pressure a government in an attempt to win the monopoly. (ii) The resources that the firms are willing to spend in an attempt to win the monopoly may possibly add up to the whole monopoly profit. (iii) The costs of these activities are wasted, so we should indeed add them to the social cost of having a monopoly.

- *X-inefficiency*: From the lecture slides (see also Tirole, p. 75): "**X-inefficiency**" (Harvey Leibenstein, 1966).

  - Under a monopoly, the firm's operations are not carried out at minimal costs, due to "managerial slack".
  - Hicks (1935) had expressed a related idea, using the famous phrase "The best of all monopoly profits is a quiet life".

b) **Show that the optimal price charged by a monopoly firm is a non-decreasing function of marginal cost. More precisely, let $C_1(q)$ and $C_2(q)$ be two cost functions for the monopolist, and**

let p_1^m and p_2^m be the associated profit-maximizing monopoly prices. Suppose that the cost functions are differentiable and that $C_2'(q) > C_1'(q)$ for all $q > 0$. Prove that then $p_1^m \leq p_2^m$.

- *Proof*: Since p_1^m is the optimal (=profit maximizing) price given $C_1(\cdot)$, a monopolist with cost function $C_1(\cdot)$ weakly prefers p_1^m to p_2^m:

$$\boxed{p_1^m q_1^m - C_1(q_1^m) \geq p_2^m q_2^m - C_1(q_2^m).} \tag{1}$$

Similarly, a monopolist with cost function $C_2(\cdot)$ weakly prefers p_2^m to p_1^m:

$$\boxed{p_2^m q_2^m - C_2(q_2^m) \geq p_1^m q_1^m - C_2(q_1^m).} \tag{2}$$

Now add (1) and (2):

$$\begin{aligned} & p_1^m q_1^m - C_1(q_1^m) + p_2^m q_2^m - C_2(q_2^m) \\ \geq\ & p_2^m q_2^m - C_1(q_2^m) + p_1^m q_1^m - C_2(q_1^m). \end{aligned}$$

Simplifying yields

$$-C_1(q_1^m) - C_2(q_2^m) \geq -C_1(q_2^m) - C_2(q_1^m)$$

Rewrite this by moving everything to left-hand side:

$$[C_2(q_1^m) - C_2(q_2^m)] - [C_1(q_1^m) - C_1(q_2^m)] \geq 0.$$

Rewrite the left-hand side, equivalently, on integral form:

$$\boxed{\int_{q_2^m}^{q_1^m} [C_2'(z) - C_1'(z)]\, dz \geq 0.} \tag{3}$$

Since we have assumed that $C_2'(q) > C_1'(q)$ for all $q > 0$, (3) implies that $q_1^m \geq q_2^m$. Moreover, since the demand function is downward-sloping, $q_1^m \geq q_2^m$ implies $p_1^m \leq p_2^m$. $\square$

c) **Consider a market with four firms. Their market shares are 10, 20, 30 and 40 percent. Calculate the Herfindahl index and the 2-firm concentration ratio for this market.**

- The Herfindahl index is defined as the sum of the squared market shares, $HI = \sum_{1=1}^{n} s_i$, where s_i is firm i's market share and n is the number of firms in the market. Therefore, the Herfindahl index for this market equals

$$HI = (0.1)^2 + (0.2)^2 + (0.3)^2 + (0.4)^2 = 0.01 + 0.04 + 0.09 + 0.16 = 0.3.$$

- The 2-firm concentration index ratio is defined as the sum of the two largest firms' market shares. Therefore this ratio equals $0.4 + 0.3 = 0.7$.

d) **Explain what is meant by "resale-price maintenance" (RPM). Also, explain verbally the Chicago argument for why RPM should not be illegal.**

- Resale-price maintenance means that the supplier of a good tells a retailer (in a contract between the two parties) what price the retailer must charge the final consumers. That is, the final price to the consumers is simply specified in the contract between the supplier and the retailer.

- The gist of the Chicago argument is that (i) there is an externality between vertically related firms and that (ii) the nature of this externality is such that also consumers, not only the firms themselves, benefit if the firms cooperate and thereby internalize the externality. Of course, this is true also for two horizontally related firms. But then the typical situation is that the goods are *substitutes*: firm 1's demand *drops* if firm 2 lowers its price, yielding an equilibrium price that is too low relative to the joint-profit maximizing price. In the vertical story, the input good and the final good are *complements*, so the externality works in the opposite direction.

- One important example of an externality between two vertically related firms is the one that arises because of so-called double marginalization. This refers to a situation where two firms charge a monopoly price "on top of each other". First a monopoly supplier sets a price that a retailer must pay, a price that equals the supplier's marginal cost plus some margin. Then the retailer, also a monopolist in its market, sets the final consumers' price, a price that equals the retailer's marginal cost (which is the same as the price set by the supplier) plus some margin. The final consumer price thus involves *two* margins on top of the supplier's marginal cost. This price will be higher than the price that would be set optimally by the two firms jointly, if the maximized the sum of the upstream- and the downstream firms' profits. Therefore also the consumers would gain if the firms cooperated (the firms are obviously benefitting from this). However, it is possible to internalize the externality between the two firms without having a full merger between the firms, but instead use RPM — that is, to introduce a clause in the contract between the two firms that stipulates which price the retailer must charge.

- That is, we can summarize the Chicago argument as follows:

  - RPM is not necessarily anti-competitive and harmful to the consumers — all parties may gain.
  - The real problem is the fact that the upstream and downstream firms have monopoly power (or, more generally, market power). RPM is just a (welfare-enhancing) by-product of that monopoly power.
  - When designing competition policy one may (if one buys the Chicago argument) want to treat vertical relationships differently from horizontal ones. In particular, the argument suggests that interaction

between vertically related firms — for example in the form of RPM — is much more likely to desirable from a welfare point of view.

e) **Explain briefly the conjectural-variations approach to modelling an oligopoly.**

- The idea is to assume that (in, say, a duopoly) the firms believe that a change in one firm's output leads to a change in the rival's output, even though the firms' choices are otherwise modelled as being simultaneous. The degree to which the rival's output changes is captured by a parameter, the conjectural variations parameter. This parameter is typically assumed to be constant (and often also identical across firms). As this parameter takes various values, the outcome of the model (the equilibrium quantities) can be made identical to, for example, the outcome under Cournot or Bertrand competition or the collusive outcome. The approach is therefore used as a reduced-form way of capturing a family of different models with different degrees of competition.

Question 2

Consider a market with two firms that produce differentiated goods. Firm 1's demand is given by $q_1 = 30 - 2p_1 - p_2$, where q_1 is firm 1's sold quantity, p_1 is firm 1's price, and p_2 is firm 2's price. Firm 2's demand is given by $q_2 = 30 - 2p_2 - p_1$, where q_2 is firm 2's sold quantity. Neither firm has any productions costs. The firms simultaneously choose their respective price, with the objective of maximizing their profits (which here are the same as their revenues). The firms interact only once.

(a) **Solve for the Nash equilibrium of the model. Calculate the equilibrium level of the prices, the quantities and the profits of each firm.**

- Firm 1's profits (which here are the same as its revenues) equal

$$\pi_1 = p_1 q_1 = p_1 (30 - 2p_1 - p_2).$$

Similarly, firm 2's profits equal

$$\pi_2 = p_2 q_2 = p_2 (30 - 2p_2 - p_1).$$

A Nash equilibrium is defined as a pair of prices such that neither firm can increase its profits by unilaterally (i.e., while the rival keeps its price fixed) choosing some other price. Therefore, a Nash equilibrium must satisfy the following first order conditions:

$$\frac{\partial \pi_1}{\partial p_1} = (30 - 2p_1 - p_2) - 2p_1 = 0$$

and

$$\frac{\partial \pi_2}{\partial p_2} = (30 - 2p_2 - p_1) - 2p_2 = 0.$$

Now subtract one first order condition from the other:

$$[(30 - 2p_1 - p_2) - 2p_1] - [(30 - 2p_2 - p_1) - 2p_2] = 0 \Leftrightarrow$$

$$3 (p_2 - p_1) = 0 \Leftrightarrow p_1 = p_2.$$

That is, if the two first order conditions are satisfied, then the prices must be the same. We can thus impose symmetry (setting $p_1 = p_2 = p$) in one of the conditions, thereby obtaining the following equilibrium price:

$$(30 - 2p - p) - 2p = 0 \Leftrightarrow p^* = \frac{30}{5} = 6.$$

The equilibrium quantity sold by each firm can now be calculated by plugging in p^* in one of the demand functions:

$$q_1^* = 30 - 2p^* - p^* = 30 - 3p^* = 30 - 18 = 12.$$

And, by symmetry,

$$q_2^* = 12.$$

Each firm's equilibrium profit can be calculated by plugging in $p^* = 6$ and $q_1^* = q_2^* = q^* = 12$ in the profit expressions:

$$\pi_1^* = p_1^* q_1^* = 6 * 12 = 72.$$

By symmetry, $\pi_2^* = \pi_1^*$.

(b) **Suppose the firms merge and that the merged firm produces and sells both goods, sold at the prices p_1 and p_2, respectively Solve for the prices that maximize the profits (= revenues) of the integrated firm. Also calculate the associated quantities and the equilibrium level of profits of the integrated firm.**

- The merged firm's profits, denoted π^m, (which here are the same as its revenues) equal the sum of the two individual firms' profits, so

$$\pi^m \equiv \pi_1 + \pi_2 = p_1 q_1 + p_2 q_2 = p_1 (30 - 2p_1 - p_2) + p_2 (30 - 2p_2 - p_1).$$

The firm is now alone on the market, and we don't need to identify a Nash equilibrium (in a non-trivial sense) but simply the merged firm's optimal prices. The optimal prices must satisfy the following first order conditions:

$$\frac{\partial \pi^m}{\partial p_1} = (30 - 2p_1 - p_2) - 2p_1 - p_2 = 0$$

and

$$\frac{\partial \pi^m}{\partial p_2} = (30 - 2p_2 - p_1) - 2p_2 - p_1 = 0.$$

Now subtract one first order condition from the other:

$$[(30 - 2p_1 - p_2) - 2p_1 - p_2] - [(30 - 2p_2 - p_1) - 2p_2 - p_1] = 0 \Leftrightarrow$$

$$2(p_2 - p_1) = 0 \Leftrightarrow p_1 = p_2.$$

That is, if the two first order conditions are satisfied, then the prices must be the same. We can thus impose symmetry (setting $p_1 = p_2 = p$) in one of the conditions, thereby obtaining the following equilibrium prices:

$$(30 - 2p - p) - 2p - p = 0 \Leftrightarrow p^{**} = p_1^{**} = p_2^{**} = \frac{30}{6} = 5.$$

The equilibrium quantity of each good sold by the merged firm can now be calculated by plugging in p^* in one of the demand functions:

$$q_1^{**} = 30 - 2p^{**} - p^{**} = 30 - 3p^{**} = 30 - 15 = 15.$$

And, by symmetry,

$$q_2^{**} = 15.$$

The merged firm's maximized profit can be calculated by plugging in $p^{**} = 5$ and $q_1^{**} = q_2^{**} = q^{**} = 15$ in the profit expression:

$$(\pi^m)^* = p_1^{**} q_1^{**} + p_2^{**} q_2^{**} = 5 * 15 + 5 * 15 = 75 + 75 = 150.$$

(c) **[You are encouraged to attempt part c) even if you have not been able to answer parts a) and b).] Compare the results under a) and b) above and explain the intuition behind any differences. Relate your answer to so-called double marginalization.**

- Summing up the results derived in parts a) and b), we have:

	Separate firms	Merged firms
Market price	6	5
Output of each good	12	15
Industry profits	144	150

The table shows that the fact that the two firms merge leads to a lower price (and thus higher output). This means that the consumers will benefit from the merger (as their utility is decreasing in the price). The table also shows that industry profits increase as a consequence of the merger, which means that not only the consumers but also the firms are better off if the merger takes place (as long as the merged unit's profits are split between the owners of the original firms in a way that ensures that they get at least as much as they would get without the merger). We can conclude that everyone in this economy – both consumers and firms — benefit from the merger. This (in particular that the consumers benefit) may sound surprising, as the merger also means that the number of firms (and in that sense, the level of competition) in the market goes down.

- The intuition behind this result can be understood by noting that the demand functions are such that the two goods are complements in the sense that consumers want to buy more of good 2 if already consuming a relatively large quantity of good 1 (all else being equal). For example, the two goods could be bread and butter. If a consumer gets access to some extra loafs of bread, then that consumer also wants some more butter (because those two goods go well together, according to the consumer's preferences). The fact that the goods are complements means that an action by one firms exerts a positive externality on the other firm; in particular, if firm 1 lowers its price (on bread, say), then that action increases the demand not only for bread but also for the demand of the other firm's good (say, of butter).

(d) **What do economists mean by "strategic substitutes" and "strategic complements"? In the original model above with two separate firms, are the two firms' choice variables strategic substitutes or strategic complements?**

- The definition of strategic substitutes and complements, respectively, is that the cross derivative of each player's payoff function is negative respectively positive. From above we have that firm 1's payoff function is

$$\pi_1 = p_1 (30 - 2p_1 - p_2).$$

Differentiating this twice, first w.r.t. p_1 and then w.r.t. p_2, yields

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = -1.$$

That is, this cross-derivative is negative. By symmetry, the cross derivative of firm 2's payoff function has the same sign. That means that the firms' choice variables in this model are strategic substitutes.

Question 3

Consider Tirole's version of the Rotemberg-Saloner model (exactly the same version as we studied in the course). In a market there are two identical firms, firm 1 and firm 2. They produce a homogeneous good and each firm has a constant marginal cost $c \geq 0$. There are infinitely many, discrete time periods t (so $t = 1, 2, 3, \ldots$), and at each t the firms simultaneously choose their respective price, p_1^t and p_2^t. The firms' common discount factor is denoted $\delta \in (0, 1)$. As the good is homogeneous, demand is a function of the lowest price, $p^t = \min\{p_1^t, p_2^t\}$. Demand is stochastic: with probability $\frac{1}{2}$, demand in period t is high, $q^t = D_H(p^t)$ (> 0); and with probability $\frac{1}{2}$, demand in period t is low, $q^t = D_L(p^t)$ — with $D_H(p^t) > D_L(p^t)$ for all p^t. Demand realizations are independent across time. If the firms charge the same price they share the demand equally between themselves.

The firms can observe the rival firm's choice of price once it has been made. Moreover, the firms can observe the current period's demand realization, before choosing their price. However, the demand realizations in future periods are not known to the firms.

a) **Let p_s^m be the state s monopoly price, i.e., the price that maximizes $(p - c) D_s(p)$. Consider (exactly as in the course) a grim trigger strategy in which each firm starts out charging the price $p_s^t = p_s^m$ if the period t state is s. However, if there has been any deviation from that behavior by anyone of the firms in any previous period, then each firm plays $p_s^t = c$.**

   (i) **Derive a (necessary and sufficient) condition for when the above trigger strategy is part of a subgame perfect Nash equilibrium. In particular, state the condition as $\delta \geq \delta_0$, where δ_0 is a function of the maximized industry profits in state s [i.e., of $\Pi_s^m \equiv (p_s^m - c) D_s(p_s^m)$] but not a function of δ.**

- We must investigate under what conditions a typical firm does not want to deviate from the trigger strategy described in the question, given that the other firm follows the trigger strategy.

- To that end, first note that, if following the equilibrium strategy when the state is s, a firm's overall payoff equals

$$\boxed{\tfrac{1}{2}\Pi_s^m + \delta V,} \tag{4}$$

where

$$V = \frac{\frac{1}{2}\frac{\Pi_L^m}{2} + \frac{1}{2}\frac{\Pi_H^m}{2}}{1-\delta} = \frac{\Pi_L^m + \Pi_H^m}{4(1-\delta)}.$$

In words, the firm will in the current period get half of the monopoly profits given state s. In the following periods the state is not yet known, so what enters as the second term of (4) is half of the the stream of expected monopoly profits, discounted to the present period.

- If making the best possible deviation (which is to just undercut the rival's price), the firm can get (almost)

$$\boxed{\Pi_s^m + 0,}$$

because from next period onwards the firm gets a zero profit according to the trigger strategy.

- That is, there is no incentive to deviate if

$$\frac{1}{2}\Pi_s^m + \delta V \geq \Pi_s^m \Leftrightarrow \boxed{\delta V \geq \tfrac{1}{2}\Pi_s^m.}$$

This condition must hold both for $s = L$ and $s = H$. Because $\Pi_H^m > \Pi_L^m$, the high-state condition is the most stringent (i.e., the tightest one). Therefore the condition holds for both states if and only if it holds for the high state:

$$\underbrace{\delta\frac{\Pi_L^m + \Pi_H^m}{4(1-\delta)}}_{\equiv \delta V} \geq \frac{1}{2}\Pi_H^m \Leftrightarrow \boxed{\delta \geq \tfrac{2\Pi_H^m}{3\Pi_H^m + \Pi_L^m} \equiv \delta_0.}$$

The last inequality is the one that we were asked to derive. The reasoning above (which investigates the incentives to deviate *on* the equilibrium path) shows that this condition is necessary for the trigger strategy to be part of an SPNE. To be able to conclude that the condition also is sufficient, we must consider the incentives to deviate off the equilibrium path — in particular, we must show that it is optimal for a firm to follow the trigger strategy when being in a punishment phase (given that the above condition is satisfied) . However, that is indeed, almost trivially, optimal, since the trigger strategy specifies that the firms should revert to the one shot Nash equilibrium (p=MC) in case of a deviation, so the firms are by construction of the trigger strategy making best replies in that situation.

1. (ii) **[You are encouraged to attempt part (ii) even if you have not been able to answer part (i).] Interpret your results under (i). When is full collusion most difficult to sustain — in a high or a low state? Explain the intuition. Also explain how the possibility of full collusion depends on δ, Π_L^m and Π_H^m and explain the intuition.**

- The key model ingredient: **demand fluctuates** stochastically (but is known when setting p). Otherwise it is a standard duopoly model with price-setting firms, interacting over an infinite horizon.

- One can, as in a standard repeated game, sustain a collusive equilibrium if the firms care sufficiently much about future profits (high enough discount factor δ). However, in this model, the requirement on the discount factor when having a high demand state is more stringent — the firms must be more patient than in the known-demand model for cooperation to be possible. The reason for this is that in the uncertainty model, in a high demand state, demand will be unusually high. The demand realization is by assumption independent over time, so the expected profits tomorrow and onwards are the same regardless of today's demand state. This means that when the demand is known to be high today, then the incentive to deviate from the equilibrium is higher than in the standard model, as the "one-period temptation" is unusually high whereas the "long-term reward of not deviating" is the same. The conclusion is that there is a tendency for collusion to break down in a high demand state (hence price war during booms and counter-cyclical prices).

- As is clear from the condition $\delta \geq \delta_0$ derived above, a larger δ makes it easier to sustain full collusion. The reason for this is the same as in the standard model, namely that the reward for cooperating (i.e., choose the collusive price) comes in the future, whereas a firm can gain in the short term by not cooperating. therefore the cooperation requires that the firm cares sufficiently much about the future (has a high δ).

- The critical value δ_0 is decreasing in Π_L^m and increasing in Π_H^m. Thus a larger Π_L^m makes collusion easier, and a larger Π_H^m makes collusion harder. The intuitive reason for these results is that when the difference between Π_L^m and Π_H^m is large, then the "one-period temptation" discussed above is also large, which makes collusion more difficult.

b) **Explain briefly what is meant by "facilitating practices".**

- From the slides:

- **"Facilitating practices"**: Practices adopted by firms that increase the likelihood of collusion.

Examples

- **Exchange of information** (e.g., publishing prices).

- **Trade associations**. Can facilitate information exchange or make punishments harsher (exclusion from the TA).

- **Most-favored customer clause.** Contractual commitment by a seller that all customers will pay the lowest price paid by any customer.

  - May make it more costly for a firm to deviate from a collusive agreement (since it has to charge the lower price to all customers).

- Gives the customers a stronger incentive to watch out for (secret) price cuts, which then also the competitors can find out about.

END OF EXAM